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MCA-003-0492002 Seat No. ________

B. Sc. / M. Sc. (Applied Physics) (Sem. II) (CBCS) Examination

April / May - 2018

Paper - 6 : Applied Mathematics

(New Course)

Faculty Code : 003
Subject Code : 0492002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures on the right indicate marks.

1 Attempt Any **Seven** short questions : (Two marks each) **14**

(1) Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$.

(2) State Newton's law of cooling.

(3) Define : Homogeneous function.

(4) If $z = x^2 y + 3xy^4$ wher $x = \sin 2t$ and $y = \cos t$, find $\frac{dz}{dt}$ where $t = 0$

(5) If $f(x, y) = x^2 + 3xy + y - 1$ then find f_x *and* f_y.

(6) Solve $\frac{\partial^2 z}{\partial x^2} = \sin x$.

(7) Solve $pq = p + q$

(8) Evaluate $\iint xy(x+y)\,dxdy$ over the area between $y = x^2$ *and* $y = x$.

(9) State Rolle's theorem.

(10) Verify Cauchy's mean value theorem for $f(x) = x^2$ *and* $g(x) = x^3$ in [1,2]

2 (A) Write answers of Any **Two** : (Five marks each) **10**

(1) Solve $\left(x^2 - y^2\right) dx - xydy = 0$

(2) Solve $xy\left(1 + xy^2\right)\frac{dy}{dx} = 1.$

(3) Solve $\left(x^2 - ay\right)dx = \left(ax - y^2\right)dy.$

(4) Solve $(px - y)(py + x) = a^2 p$

(B) Write answers of Any **Two** : (Two marks each) **4**

(1) If $x\frac{dy}{dx} + y = x^3 y^6$ then find I.F.

(2) Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2.$

(3) Solve $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$

(4) State Kirchhoff's law of electric circuits.

3 (A) Write answer of Any **Two** : (Five marks each) **10**

(1) State and prove Euler's theorem on homogeneous functions.

(2) If $u = \ln\left(x^3 + y^3 + z^3 - 3xyz\right)$ then prove that $\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial y} + \frac{\partial}{\partial y}\right)^2 u = -\frac{9}{(x+y+z)^2}.$

(3) If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ then find $J\left(\frac{x, y, z}{r, \theta, \phi}\right)$

(4) Find the stationary points of the function $f(x, y) = x^2 y - xy^2 + 4xy - 4x^2 - 4y^2.$

(B) Write answers of Any **Two** : (Two marks each) **4**

(1) Define : Partial derivative.

(2) Find $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1.$

(3) Define : Stationary point and Saddle point.

(4) Define : Jacobian.

4 (A) Write answers of Any **Two** : (Five marks each) **10**

(1) Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \cos(2x + 3y)$.

(2) Solve $\dfrac{y^2 z}{x}\dfrac{\partial z}{\partial x} + xz\dfrac{\partial z}{\partial y} = y^2$

(3) Solve $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$, given $u(x,0) = 6e^{-3x}$ using the separation of variables method.

(4) Solve $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin 2y$

(B) Write answers of Any **Two** : (Two marks each) **4**

(1) Solve $\dfrac{\partial^2 z}{\partial x^2} = a^2 \dfrac{\partial^2 z}{\partial y^2}$

(2) Define : Lagrange's linear equation.

(3) Eliminate the arbitrary function from the equation $z = xy + f\left(x^2 + y^2\right)$

(4) Solve $(p + q)(z - xp - yq) = 1$

5 (A) Write answer of Any **Two** : (Five marks each) **10**

(1) State Lagrange's mean value theorem. Hence apply Lagrange's theorem to $f(x) = x^3 + 4x \ in \ [-1,1]$.

(2) Evaluate the following integrals

(i) $I_1 = \displaystyle\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x + y + z)\, dxdydz$

(ii) $I_1 = \displaystyle\int_{0}^{1}\int_{y^2}^{1}\int_{0}^{1-x} x\,dxdydz$

(3) Evaluate $\iiint (x+y+z)\, dxdydz$ over the tetrahedron bounded by planes $x=0, y=0, z=0$ and $x+y+z=1$.

(4) Expand the following using Taylor's theorem

(i) $f_1(x)=\cos x$ about $x=1$

(ii) $f_2(x)=e^x$ about $x=0$

(iii) $f_3(x)=\log(1+x)$ about $x=1$

(B) Write answers of Any **Two** : (Two marks each) **4**

(1) Find area lying between the parabola $y=x^2$ and line $x+y-2=0$.

(2) Change the order of integration for

$$I=\int_0^1\int_3^{3y} e^{x^2}\, dxdy$$ and evaluate it.

(3) Evaluate : $\int_0^5\int_0^{x^2} x\left(x^2+y^2\right) dxdy.$

(4) State Taylor's theorem.